Development of Web-Based Engineering Numerical Software (WENS) Using MATLAB: Applications to Linear Algebra

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Received 18 February 2003; accepted 28 April 2003

ABSTRACT: In this article, we present Web-based Engineering Numerical Software (WENS) developed using MATLAB and its Web Server Toolbox (WST) in order to provide basic solution tools in linear algebra, including eigenvalue problems, which are extensively used in engineering applications and education. © 2003 Wiley Periodicals, Inc. Comput Appl Eng Educ 11: 67–74, 2003; Published online in Wiley InterScience (www.interscience.wiley.com); DOI 10.1002/cae.10038

Keywords: web-based education; distance learning; linear algebra; eigenvalue problems; MATLAB Web Server Toolbox

INTRODUCTION

Engineering is the application of science, including mathematics, physics, chemistry, biology, and so on, to the optimum conversion of natural resources for human use. Basic knowledge and understanding of science is thus inevitable in the undergraduate as well as graduate curriculum of engineering. This learning procedure starts with applying fundamental principles or well-known governing equations to understand and utilize natural and/or engineering phenomena. In many engineering courses, students are required to solve problems with the tools of applied mathematics and then analyze the calculation results to extract useful engineering implications. Except for very simple and ideal cases that are used as instructional examples in classes, most realistic engineering problems, especially design-oriented ones, must be solved numerically since analytic solutions are often too

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complicated. Therefore, an undergraduate engineering curriculum typically includes several courses on numerical analysis. As a branch of applied mathematics, numerical analysis connects fundamental science with practical engineering, requiring relevant education of computer programming languages, such as FORTRAN [1,2], Pascal [3], C [4], etc., to implement computational algorithms. It can be very time intensive, however, to teach and implement these programming languages and algorithms. Instead, such high level languages as MATLAB, MAPLE, and MATHEMATICA provide friendlier user-interfaces by exploiting pre-existing numerical libraries, allowing users to avoid complicated coding. Among those languages available, MATLAB along with the accompanying toolbox(es) is extensively used for engineering research as well as for education [5–8].

Accreditation Board of Engineering and Technology (ABET) released the Criteria for Accrediting Engineering Programs [9] for the 2002–2003 accreditation cycle. The criteria indicate several points related to math/science education and the corresponding reflections:

“Engineering programs must demonstrate that graduates have (i) an ability to apply knowledge of mathematics, science, and engineering; (ii) an ability to identify, formulate, and solve engineering problems; and (iii) an ability to use the techniques, skills, and modern engineering tools necessary for engineering practice.”

The criteria emphasize not only the first two items related to knowledge and problem-solving skills in a traditional sense, but also modern engineering tools, which must include the use of (1) computers and software, and (2) Internet communications. These two tools are often combined and utilized in web-based environments for undergraduate education in science [10], engineering [11,12], and health/medical fields [13,14]. The foremost purpose of the current project, named WENS, is therefore to provide undergraduate engineering courses with basic resources of applied mathematical tools via the World Wide Web (WWW).

BACKGROUND AND OBJECTIVES

In civil and environmental engineering, the undergraduate curriculum has many courses in which instructors need to use basic solution tools in applied mathematics, mainly those in linear algebra. Examples of those courses include, but are not limited to, finite difference and finite element methods, structural dynamics/analysis, hydrologic processes, ground water modeling, and contaminant transport. In some cases, different instructors in different courses redundantly teach the same numerical methods, and the instructors tend to choose mathematical software of their preference to help the students solve the problems. Accordingly, there have been increasing needs among instructors as well as students for common resources of solution tools to minimize this redundancy in learning mathematical solution tools. Students can often use math software packages, which are usually installed on UNIX workstations in computer labs. The use of computer labs by the undergraduate students, however, can be hampered by limited lab operation hours and space. Remote access from students’ residences or elsewhere may not be very convenient, either: students would need an X-window emulator for their Windows-based PCs or Macintoshes to perform graphics-related works using the Workstations. Trial versions of X-window emulators are available from several vendors free of charge, but restrictions normally apply to the trial period (usually 15–30 days) and/or operation time (mostly 30 min). Instead, web-based software can be a good replacement as it can provide students with free accessibility to numerical tools if an Internet connection is available. Instructors can also benefit by minimizing the time spent for teaching extra math software. It is also worth noting that the number of Internet-connected PCs on campus is much larger than that of UNIX workstations. These observations have motivated us to develop Web-based Engineering Numerical Software (WENS) using MATLAB and its Web Server Toolbox (WST). The possible reflection is that engineering (undergraduate) students are often more accustomed to the application of mathematics through the easy and convenient use of WENS. Although this software cannot be an ultimate math-solving tool for all engineering purposes, students can be encouraged to expand their experiences in scientific/engineering calculations by using WENS.

METHOD

After an intensive search to decide on the programming language that can be embedded on the WWW, two options remained: Java and MATLAB. Java, released by Sun Microsystems in March 1995, is a platform-independent object-oriented language used to create applets for the WWW as well as for stand-alone applications. While the Hyper Text Markup Language (HTML) is used to create static web pages using tag building blocks, or objects, Java allows programmers
to inspect and manipulate those objects to punch up static pages with intelligence, interactivity, and simple animations. A standard version of Java in consensus among vendors, developers, and users, however, does not seem to have appeared yet since many vendors like Sun Microsystems, Microsoft, and Borland have been releasing Java compilers with their own specific features. Some of the features supported in an old Java version are not often included in a newest version. This implies that a new version of Java could be incompatible with old scripts, although the Java language itself provides great advantages and good working environments to developers and users. On the other hand, in MATLAB (Mathworks, Inc., Natwick, MA) consistency and flexibility are maintained among most pre-released and current versions. In addition to the basic kernel, scientific applications have been classified professionally as “Toolboxes,” which contain necessary math/graphic functions for each application. Therefore, using MATLAB allowed us to focus on designing educational web-applications without developing necessary numerical as well as graphical libraries: this is the main reason why we selected MATLAB instead of Java, despite Java’s better web-based design features.

We purchased the basic MATLAB kernel, Extended Symbolic Toolbox (EST), and WST; and investigated how MATLAB WST works to link inputs from web site visitors with the kernel [15,16]. Demo scripts included in WST were helpful for basic understanding, as were personal communications with Mathworks, Inc. After the structure of WST was figured out, planned contents of the software were reviewed and implemented. The final product is now available on our project website (http://wens.eng.hawaii.edu).

DEVELOPED PROGRAMS

Figure 1 illustrates how MATLAB WST provides an interface between web users and the MATLAB kernel. WST receives user input through HTTP daemon platformed in Red Hat Linux 7.2 operating system, and sends the input data and the name of the designated M-file, which contains necessary MATLAB scripts, to the kernel. The kernel calls on the M-file to perform ordered calculations with obtained user input. The final results are displayed in a new web page by the kernel through WST.

Based on the above procedure, we developed the following applications: (1) linear algebra applications for single matrix property, multiple matrix operation, and linear system solving; (2) application to structural engineering; and (3) symbolic calculations.

**Linear Algebra**

Basics of linear algebra are briefly introduced and the usage of applications is explained by simple examples.

**Single Matrix Property.** This section is designed to investigate the properties of a square \((n \times n)\) matrix \(A\) such as rank, determinant, square \((A^2)\), transpose, inverse, eigenvalues, and eigenvectors. The input format of the square matrices is identical to that used in MATLAB console. For each calculation, a drop-down box of [yes, no] option with ‘yes’ default, is embedded so that users can obtain the only necessary results thereby saving computational resources.

**Double Matrix Operation.** Two input textboxes are embedded to receive the elements of two square matrices. Basic operations with the two matrices, \(A\) and \(B\), are addition \((A + B)\), subtraction \((A - B)\), multiplication \((AB)\), (so called) division \((AB^{-1})\), and user-defined operation. As in the case of “single matrix operation,” optional drop-down boxes allow the user to select the desired operations. In “user’s input,” the user can specify a formula in terms of \(A\) and \(B\) (e.g., \(A^2 + B^2 + 3A + 2B\)), for which the corresponding MATLAB format is \(A^2 + B^2 + 3*A + 2*B\), submit it, and obtain the results in a new output window. Most of the usual MATLAB commands for matrix operation can be used in the “user’s input” box.
**Linear System.** In this application, linear systems \((Ax = b)\) can be solved, where \(A\) and \(b\) are a square and a column matrix, respectively, and \(x\) is the unknown matrix having the same size as \(b\). The elements of \(A\) and \(b\) should be typed in by the user, and the solution vector \(x\) is displayed in a new web page of the user’s browser. The intent of this application is mainly to provide the solution of a linear equation using a generic Gauss elimination procedure, of which the MATLAB command is \(x = A\backslash b\). Thus, the selection of the appropriate numerical scheme based on the matrix structure, such as tridiagonal, banded, symmetric, etc., is not included.

**Application to Structural Engineering**

Structural Dynamics I is a graduate-level dynamics course taught in the Department of Civil and Environmental Engineering, University of Hawaii (UH) at Manoa. This course is concerned with, among other things, the classical modal analysis of multi-degree-of-freedom systems arising in problems of structural engineering. The modal analysis technique employs the eigenvalues and the corresponding eigenvectors of the system under consideration to determine the response time history. Thus, solving the eigenvalue problem is the crucial step in the overall solution procedure. Although this is a graduate course, we believe it is worth using WENS to help students understand the basic concepts and do homework accordingly.

A multi-story building of typical configuration is frequently modeled as a *shear building*, for which it is assumed that the mass is concentrated at the floor levels; axial deformation is neglected; the lateral stiffness is provided by the flexural stiffness of the columns only; and the floors are rigid [17]. With these assumptions, the story displacements at the floor levels become the only degrees of freedom of the system, with the number of degrees-of-freedom equal to the number of stories. For example, a four-story framed structure modeled as a shear building is shown in Figure 2, where \(u_i\) and \(m_i\) represent the lateral displacement and concentrated mass at the \(i^{th}\) floor level, respectively, while \(k_i\) is the lateral stiffness of the columns in the \(i^{th}\) story. Based on these system characteristics, the mass and stiffness matrices, \(M\) and \(K\), respectively, can be constructed, and the substitution of the matrices into the equations of motion leads to the generalized eigenvalue problem:

\[
Kx = \lambda Mx
\]

(1)

Here, \(\lambda\) is the eigenvalue, interpreted as the square of the natural vibration frequency \(\omega\), and \(x\) is the corresponding eigenvector representing the vibrational shape of the mode associated with the vibration frequency.

In the MATLAB code developed for WENS, the Rayleigh’s quotient iteration procedure [17] was used to solve the eigenvalue problem. In this procedure, students are asked to provide the initial estimate of the eigenvalue to start the iteration; the initial estimate for the eigenvector is, for simplicity, set to a vector for which all components are equal to one. The estimate for the smallest eigenvalue can be made based on conventional knowledge regarding the fundamental vibration period of a multi-story building. After the first eigenvalue is found, larger estimates are used to find the subsequent eigenvalues until all eigenvalues are obtained. Students are advised that the total number of eigenvalues is equal to the number of stories, and different eigenvalues are well separated from each other for a typical building structure like the one shown in Figure 2.

Figure 3a shows the inputs screens of WENS for the analysis case, for which

\[
K = \frac{k}{9} \begin{bmatrix} 16 & -7 & 0 & 0 \\ -7 & 12 & -5 & 0 \\ 0 & -5 & 8 & -3 \\ 0 & 0 & -3 & 3 \end{bmatrix}
\]

(2)

and

\[
M = m \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{3}{4} & 0 & 0 \\ 0 & 0 & \frac{3}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{bmatrix}
\]

(3)

Figure 2 Shear building model for multi-story building.
where \( k = 350000 \text{ N/m} \) and \( m = 400 \text{ kg} \). Using an initial estimate of the eigenvalue of \( \lambda = 100 \), WENS calculates the natural vibration frequency of \( \omega = 11.1208 \) from the smallest eigenvalue of \( \lambda = 123.673 \), and the corresponding eigenvector shown in Figure 3b.

### Symbolic Calculation

The symbolic calculation implemented in this section is the differentiation of a user-provided analytic function: if a user wants to differentiate a function

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**Structural Application**

It takes only the following these three easy steps to obtain an eigenpair for a generalized eigenvalue problem in structural dynamics:

1. Type in the Stiffness (K) and mass (M) matrices, putting spaces or commas between elements and separating rows by semicolons.
   - For example, if you want a 4 by 4 square matrix, you can type in like the following:
     
     \[
     K = \begin{pmatrix}
     16 & -7 & 0 & 0 \\
     -7 & 12 & -5 & 0 \\
     0 & -5 & 8 & -3 \\
     0 & 0 & -3 & 3 \\
     \end{pmatrix}
     \]

   or

   \[
   M = \begin{pmatrix}
   1 \times 350000/9 & -7 \times 350000/9 \\
   -7 \times 350000/9 & 12 \times 350000/9 \\
   0 & -5 \times 350000/9 & 8 \times 350000/9 \\
   0 & -3 \times 350000/9 & -3 \times 350000/9 & 3 \times 350000/9 \\
   \end{pmatrix}
   \]

2. Type in your "guessed" eigenvalue as "lambda." For example, you can put "1" here (without quotes)
3. Click on the "Submit" button ("Reset" will clear the text box).

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![Figure 3](image-url)

Figure 3  (a) Captured image of the Input Screen for Structural Application of WENS web site. (b) Output Screen of WENS for the results of the Input in (a).
b

Results of Structural Application

\[ x^3 + 2x^2 + 3x, \]

he or she needs to type in the formula in a MATLAB script-form, which is \( x^3 + 2x^2 + 3x, \)

and should specify the independent variable (x in this case) for the differentiation and the order of differentiation. The result of the calculation, \( n^{th} \) order derivative, is then available in a new web page.

**DISCUSSION**

In this section, several important issues are discussed: applicability of MATLAB, student survey, distance learning, system security, and future works.

**Applicability of MATLAB**

To the best of our knowledge, the present work is the first application that connects MATLAB with the WWW through WST for an educational purpose. Another high-level mathematics software, Mathematica (Wolfram, Research, Inc., Champaign, IL), has a special product, web Mathematica, which offers access to specific Mathematica applications through a web browser or other web clients. Fisher and Michiels [12] used this application for electrical engineering education. This feature, however, is an implementation of Mathematica console onto the browser, thus restricting the development of user-friendly interface for specific engineering applications. Although the applications implemented in the current version of WENS are relatively basic and simple from the viewpoint of web-based programming, it is worth noting that the present work has laid a foundation to develop comprehensive engineering-oriented design applications via the Internet in the future.

**Student Survey**

We conducted a user survey to evaluate the pedagogical effectiveness of WENS. A survey questionnaire was distributed to the students who were taking Structural Dynamics I. By the time the survey was conducted, the students had completed an assignment, in which they solved standard and generalized eigenvalue problems using the “Single Matrix Property” and “Structural Application” capabilities of WENS, respectively.

The questionnaire consisted of the following statements: (a) WENS provides a convenient means to do matrix computations; (b) the screen compositions of the web pages are well designed; and (c) it is easy to move around different menus and to submit computing jobs. The students were asked to provide five-point-scale responses (strongly agree = 5, agree = 4, neither agree nor disagree = 3, disagree = 2, and strongly disagree = 1) to these statements. In addition, the students were invited to describe what they think good and bad parts of WENS as well as any other comments.

As indicated in Table 1, the students generally agreed that WENS provides a convenient environment in which matrix computations can be done. It was also confirmed that WENS is fairly easy to use in the sense of moving around different menus and submitting matrix computation jobs. The web design of WENS did not receive very favorable responses, which was then reflected in subsequent updates. The best part of WENS turned out to be, as intended, its user-friendly interface and easy accessibility.

**Distance Learning**

We believe that the resources available as an outcome of this work can help students (especially undergraduate) solve applied math problems frequently.

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**Table 1** Survey Results

<table>
<thead>
<tr>
<th>Statement no.</th>
<th>Strongly agree (5)</th>
<th>Agree (4)</th>
<th>Neither agree nor disagree (3)</th>
<th>Disagree (2)</th>
<th>Strongly disagree (1)</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>43%</td>
<td>57%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>4.4</td>
</tr>
<tr>
<td>(b)</td>
<td>14%</td>
<td>43%</td>
<td>43%</td>
<td>0%</td>
<td>0%</td>
<td>3.7</td>
</tr>
<tr>
<td>(c)</td>
<td>29%</td>
<td>71%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>4.3</td>
</tr>
</tbody>
</table>
arising in engineering applications via the Internet. On May 27, 1998, the office of the vice president for planning and policy at UH prepared the Revised Executive Policy E5.204, Distance Learning Plans, Policies, and Procedures [18]. It says

"The primary goal for distance learning at the University of Hawaii is to provide a window of opportunity to the rich array of quality instructional resources available to on-campus students to students anywhere in the state who are committed to higher education but are unable to attend the UH campus offering their program of choice. Another goal of distance learning is to increasingly strengthen this focus and the University's leadership role in bridging East and West. Distance learning has the potential to support the entrepreneurial goals of the University as well as to support the University’s mission beyond state boundaries."

The goals of distance learning promulgated by UH were considered when this work was proposed, and we believe that WENS, which suitably meets the goals, can significantly contribute to Distance Learning.

System Security

The Apache web server on top of the Red Hat Linux 7.2 platform provided us with a stable operating environment for performing WST of MATLAB. Although configuration of the Apache daemon may not be an easy task for most common end-users and/or software developers who do not have system administration experiences, sufficient documentation was freely available in Internet communities, which were very helpful to build a web server. One might think that it is easier and more convenient to use Microsoft Internet Information Services (MS IIS), but its Internet security is still not well guaranteed. During the project period, we tested MS IIS as a web server on Microsoft Windows 2000 operating system, but met several troubles based on remote vulnerabilities. Therefore, Linux was finally chosen as the platform. To keep the highest level of system security in the Linux platform, we closed most of the network ports except for HTTP and Secure Shell (SSH), and updated necessary patches using an automatic update agent of Red Hat Linux, called ‘up2date,’ after we registered with Red Hat Network (RHN). The most important program we patched for system security is Secure Sockets Layer (SSL) [19].

Future Works

One of the major future works for engineering education is “plotting and data fitting.” The programming of basic 2-D plotting application has already been completed. However, graphic functions of MATLAB WST revealed a network security problem. Virtual Network Computing (VNC) should be used for graphic display in web pages but, unfortunately, this VNC server is known to be remotely vulnerable. Therefore, a more secure connection combining VNC and SSH is being considered, in which membership management can be implemented to grant access to only authorized users. Resolving this network problem will provide us with a secure environment for graphical programming using pre-built and embedded graphic libraries in the MATLAB kernel and its toolboxes.

CONCLUSIONS

This article presented the development of WENS for undergraduate engineering education. It was very successful to construct a web server with RH Linux OS to implement MATLAB and the two toolboxes, i.e., EST and WST. Teaching applied mathematics, especially linear algebra, via the Internet has proven to be feasible through WENS. Freely available remote access to WENS web site can provide engineering students with a convenient educational environment. Both students and instructors who are pursuing Distance Learning can benefit using the user-friendly interface of WENS for applied mathematics education. Development of graphical application using WST needs more investigation to remove the remote vulnerability of VNC in WST. However, it is very clear that many useful features in the education of applied mathematics can be included to extend its applications in the near future, and it would thus appeal to students more.

ACKNOWLEDGMENTS

This work was supported by The President’s Educational Improvement Fund Award (2001–2002) from Interim Vice Chancellor for Academic Affairs, University of Hawaii at Manoa, and partially by Research and Training Revolving Fund Award (2002–2003) from University Resource Council, University of Hawaii at Manoa.

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Si-Hwan Park received his BS and MS degrees in civil engineering from Seoul National University, Korea, in 1993 and 1995, respectively, and the PhD degree in civil engineering from the University of Texas at Austin in 2000. Since 2001, he has been an assistant professor in the Department of Civil and Environmental Engineering at the University of Hawaii at Manoa. His research interests include wave-propagation computations involving unbounded media and numerical modeling of cohesive-frictional materials.