CEE 271: Applied Mechanics II, Dynamics
– Lecture 1: Ch.12, Sec.1-3h –

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INTRODUCTION (Sec. 12.1) & RECTILINEAR KINEMATICS (Sec. 12.2): CONTINUOUS MOTION

Today’s objectives: Students will be able to

1. Find the kinematic quantities (position, displacement, velocity, and acceleration) of a particle traveling along a ______ path.

In-class activities:

- Relations between \( s(t) \), \( v(t) \), and \( a(t) \) for ______ motion.
- Relations between \( s(t) \), \( v(t) \), and \( a(t) \) when acceleration is ______.
- Concept Quiz
- Group Problem Solving
- Attention Quiz
1. In dynamics, a particle is assumed to have
   (A) both translation and rotational motions
   (B) only a mass
   (C) a mass but the size and shape cannot be neglected
   (D) no mass or size or shape, it is just a point
   ANS: [B]

2. The average speed in a rectilinear system is defined as
   (A) $\Delta r / \Delta t$ ($\Delta r = $ displacement)
   (B) $\Delta s / \Delta t$ ($\Delta s =$ length change)
   (C) $s_T / \Delta t$ ($s_T =$ total length)
   (D) None of the above.
   ANS: [C]
APPLICATIONS

- The motion of large objects, such as rockets, airplanes, or cars, can often be analyzed as if they were _______. Why?

- If we measure the altitude of this rocket as a function of time, how can we determine its _______ and _______?
• A sports car travels along a **straight** road.
• Can we treat the car as a particle?
  ANS: __

• If the car accelerates **at a constant rate**, how can we determine its position and velocity at some instant?
An Overview of Mechanics

_________: The study of how bodies react to _____ acting on them.

1. Statics: The study of bodies in __________.
2. Dynamics
   1. __________ - concerned with the ______ aspects of motion
   2. ______ - concerned with the ____ causing the motion
A particle travels along a ______-line path defined by the coordinate axis $s$.

The position of the particle at any instant, relative to the origin, $O$, is defined by the ________, or the ________. Scalar $s$ can be positive or negative. Typical units for $r$ and $s$ are meters (m) or feet (ft).

The _________ of the particle is defined as its change in position. (Vector form: _________ and Scalar form: _________)

The _________ traveled by the particle, $s_T$, is a positive scalar that represents the total length of the path over which the particle travels.
VELOCITY

- *Velocity* is a measure of the change in the *position* of a particle. It is a vector quantity having both magnitude and direction.

- The magnitude of the velocity is called *speed*, with units of *m/s* or *ft/s*.

- The *average velocity* of a particle during a time interval (or elapsed time) $\Delta t$ is

$$ v_{avg} = \frac{\Delta s}{\Delta t} $$

(1)
VELOCITY (Cont’d)

• The ________ velocity is the time-derivative of position:

\[ v = \frac{dr}{dt} \]  \hspace{1cm} (2)

• ____ is the magnitude of velocity:

\[ v = \sqrt{v \cdot v} \]  \hspace{1cm} (3)

• __________ is the total distance traveled divided by elapsed time:

\[ (v_{sp})_{avg} = \]  \hspace{1cm} (4)
ACCELERATION

- **Acceleration** is the rate of change in the velocity of a particle. It is a vector quantity. Typical units are m/s\(^2\) or ft/s\(^2\).

- The instantaneous acceleration is the time derivative of velocity.

- Vector form:

\[
\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{r}}{dt^2} \quad (5)
\]

- Scalar form:

\[
\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d\mathbf{s}}{dt^2} = \sqrt{\mathbf{a} \cdot \mathbf{a}} \quad (6)
\]
ACCELERATION (Cont’d)

• Acceleration can be
  1. _______ (speed increasing) or
  2. negative (speed _______).

• As the book indicates, the derivative equations for velocity and acceleration can be manipulated to get

\[ = \] 

(7)

• Derive Eq. (7) using \( a = \frac{dv}{dt} \) and \( v = \frac{ds}{dt} \).
SUMMARY OF KINEMATIC RELATIONS: RECTILINEAR MOTION

• Differentiate position to get velocity and acceleration.

\[ \mathbf{v} = \frac{d\mathbf{r}}{dt} \]  \hspace{1cm} (8)

\[ \mathbf{a} = \frac{d\mathbf{v}}{dt} \]  \hspace{1cm} (9)

\[ \mathbf{a} = \mathbf{v} \frac{d\mathbf{v}}{ds} \]  \hspace{1cm} (10)

• Integrate acceleration for velocity and position.

\[ \int_{s_0}^{s} ds = \int_{0}^{t} \]  \hspace{1cm} (11)

\[ \int_{v_0}^{v} dv = \int_{0}^{t} \]  \hspace{1cm} (12)

\[ \int_{v_0}^{v} vdv = \int_{s_0}^{s} \]  \hspace{1cm} (13)

where \( s_o \) and \( v_o \) are the initial position and velocity at \( t = 0 \).
CONSTANT ACCELERATION

• The three kinematic equations can be integrated for the special case when acceleration is \( a_c \) to obtain very useful equations.
• A common example of constant acceleration is gravity; i.e., a body freely falling toward earth.
• In this case, \( a_c = g = 9.81 \text{ m/s}^2 = 32.2 \text{ ft/s}^2 \) downward. These equations are:

1. \( \int_{v_0}^{v} dv = \int_{0}^{t} a_c dt \) yields
2. \( \int_{s_0}^{s} ds = \int_{0}^{t} v dt \) yields
3. \( \int_{v_0}^{v} vdv = \int_{s_0}^{s} a_c ds \) yields
EXAMPLE

• Given: A particle travels along a straight line to the right with a velocity of \( v = (4t - 3t^2) \) m/s where \( t \) is in seconds. Also, \( s = 0 \) when \( t = 0 \).

• Find: The position and acceleration of the particle when \( t = 4 \) s.

• Plan:
  1. Establish the positive coordinate, \( s \), in the direction the particle is traveling.
  2. Since the velocity is given as a function of time, take a derivative of it to calculate the acceleration.
  3. Conversely, _______ the velocity function to calculate the position.
EXAMPLE (continued)

Solution:

1. Take a derivative of the velocity to determine the acceleration.

\[ a = \frac{dv}{dt} = \frac{d(4t - 3t^2)}{dt} = \]

\[ a = -20 \text{ m/s}^2 \quad (15) \]

(or in the ← direction) when \( t = 4 \) s

2. Calculate the distance traveled in 4s by integrating the velocity using \( s = o \):

\[ v = \frac{ds}{dt} \Rightarrow \]

\[ \int_{s_0}^{s} ds = \int_{0}^{t} (4t - 3t^2)dt \Rightarrow \]

\[ \Rightarrow s - 0 = 2(4)^2 - (4)^3 \Rightarrow \]
CONCEPT QUIZ

1. A particle moves along a horizontal path with its velocity varying with time: \( v = 3 \text{ m/s} \) at \( t = 2 \text{ s} \) and \( v = -5 \text{ m/s} \) at \( t = 7 \text{ s} \). The average acceleration of the particle is

   (A) 0.4 m/s\(^2\) →
   (B) 0.4 m/s\(^2\) ←
   (C) 1.6 m/s\(^2\) →
   (D) 1.6 m/s\(^2\) ←

   ANS: (D)

2. A particle has an initial velocity of 30 ft/s to the left. If it then passes through the same location 5 seconds later with a velocity of 50 ft/s to the right, the average velocity of the particle during the 5 s time interval is .

   (A) 10 ft/s→
   (B) 40 ft/s→
   (C) 16 ft/s→
   (D) 0 ft/s

   ANS:
GROUP PROBLEM SOLVING

• Given: Ball \( A \) is released from rest at a height of 40 ft at the same time that ball \( B \) is thrown upward, 5 ft from the ground. The balls pass one another at a height of 20 ft.

• Find: The speed at which ball \( B \) was thrown ______.

• Plan: Both balls experience a constant downward acceleration of 32.2 \( \text{ft/s}^2 \) due to gravity. Apply the formulas for constant acceleration, with \( a_c = -32.2 \text{ ft/s}^2 \).
GROUP PROBLEM SOLVING (continued)

Solution:

(1) First consider ball $A$. With the origin defined at the ground, ball $A$ is released from rest ($(v_A)_o = 0$) at a height of 40 ft ($(s_A)_o = 40$ ft). Calculate the time required for ball $A$ to drop to 20 ft ($s_A = 20$ ft) using a position equation.

\[ s_A = (s_A)_o + (v_A)_o t + \frac{1}{2} (a_c) t^2 \]  \hspace{1cm} (16)

\[ 20 \text{ft} = + ( \ ) (t) + \frac{1}{2} ( \ ) (t^2) \]  \hspace{1cm} (17)

\[ t = \]  \hspace{1cm} (18)
GROUP PROBLEM SOLVING (continued)

Solution:

(2) Now consider ball $B$. It is throw upward from a height of 5 ft ($s_B^o = 5$ ft). It must reach a height of 20 ft ($s_B = 20$ ft) at the same time ball $A$ reaches this height ($t = 1.115$ s). Apply the position equation again to ball $B$ using

\[ s_B = (s_B)^o + (v_B)^o t + \frac{1}{2}(a_c)t^2 \]  
(19)

\[ 20 \text{ ft} = 5 + (1.115) + \frac{1}{2}(-32.2)(1.115)^2 \]  
(20)

\[ (v_B)^o = \]  
(21)
ATTENTION QUIZ

1. A particle has an initial velocity of 3 ft/s to the left at $s_0 = 0$ ft. Determine its position when $t = 3$ s if the acceleration is 2 ft/s$^2$ to the right.
   - (A) 0 ft
   - (B) 6 ft ←
   - (C) 18 ft →
   - (D) 9 ft →
   Ans: (A)

2. A particle is moving with an initial velocity of $v = 12$ ft/s and constant acceleration of 3.78 ft/s$^2$ in the same direction as the velocity. Determine the distance the particle has traveled when the velocity reaches 30 ft/s.
   - (A) 50 ft
   - (B) 100 ft
   - (C) 150 ft
   - (D) 200 ft
   Ans: (B)
Today’s objectives: Students will be able to

1. Determine **position**, **velocity**, and **acceleration** of a particle using graphs.

In-class activities:

- Reading Quiz
- Applications
- \( s - t, v - t, a - t, v - s \), and \( a - s \) diagrams
- Concept Quiz
- Group Problem Solving
- Attention Quiz
1. The slope of a $v - t$ graph at any instant represents instantaneous
   (A) velocity.
   (B) acceleration.
   (C) position.
   (D) jerk.
   ANS: (B)

2. Displacement of a particle in a given time interval equals the area under the graph during that time.
   (A) $a - t$
   (B) $a - s$
   (C) $v - t$
   (D) $s - t$
   ANS:
APPLICATIONS

• In many experiments, a velocity versus position \((v - s)\) profile is obtained.

• If we have a graph for the tank truck, how can we determine its acceleration at position \(s = 1500\) feet?
ERRATIC MOTION (Section 12.3)

- Graphing provides a good way to handle complex motions that would be difficult to describe with formulas.
- Graphs also provide a visual description of motion and reinforce the calculus concepts of _________ and _________ as used in dynamics.
- The approach builds on the facts that _____ and differentiation are linked and that integration can be thought of as finding the ___ under a curve.
**$S - T$ GRAPH**

- Plots of position vs. time can be used to find velocity vs. time curves. Finding the ____ of the line tangent to the motion curve at any point is the velocity at that point, or

  \[ v = \frac{ds}{dt} \]

- Therefore, the ____ graph can be constructed by finding the slope at various points along the ____ graph.
**$V - T$ GRAPH**

- Plots of velocity vs. time can be used to find _______ vs. time curves. Finding the slope of the line tangent to the velocity curve at any point is the acceleration at that point, or

  \[ a = \frac{dv}{dt} \]

- Therefore, _____ graph can be constructed by finding the slope at various points along the __________ graph.

- Also, the distance moved (__________) of the particle is the area under the ____ graph during time ___.

\[ a_0 = 0 \]

\[ a_1 = \frac{dv}{dt} |_{t_1} \]

\[ a_2 = \frac{dv}{dt} |_{t_2} \]

\[ a_3 = \frac{dv}{dt} |_{t_3} \]
A – T GRAPH

• Given the acceleration vs. time or \( a - t \) curve, the change in velocity (\( \Delta v \)) during a time period is the area under the \( a - t \) curve.

• So we can construct a \( v - t \) graph from an \( a - t \) graph if we know the initial velocity of the particle.
A – S GRAPH

• A more complex case is presented by the acceleration versus position or ____ graph. The ____ under the ____ curve represents the change in velocity

\[
\int_{s_0}^{s_1} a \, ds = \int_{v_0}^{v_1} v \, dv =
\]

In other words, __________

• This equation can be solved for ___, allowing you to solve for the velocity at a point. By doing this repeatedly, you can create a plot of velocity versus distance (____).
Another complex case is presented by the velocity vs. distance or graph. By reading the velocity \( v \) at a point on the curve and multiplying it by the slope of the curve (\( \frac{dv}{ds} \)) at this same point, we can obtain the acceleration at that point.

\[
a = v \left( \frac{dv}{ds} \right)
\]

Thus, we can obtain an plot from the curve.
End of the Lecture

Let Learning Continue