CEE 271: Applied Mechanics II, Dynamics
– Lecture 17: Ch.15, Sec.2–4 –

Prof. Albert S. Kim

Civil and Environmental Engineering, University of Hawaii at Manoa

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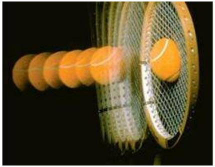
PRINCIPLE OF LINEAR IMPULSE AND MOMENTUM AND CONSERVATION OF LINEAR MOMENTUM FOR SYSTEMS OF PARTICLES

Today's objectives: Students will be able to

1. Apply the principle of linear impulse and momentum to ***a system of particles***.
2. Understand the conditions for ***conservation*** of momentum.



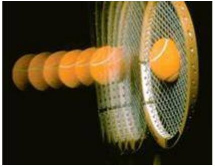
In-class activities:

- Reading Quiz
- Applications
- Linear Impulse and Momentum for a System of Particles
- Conservation of Linear Momentum
- Concept Quiz
- Group Problem Solving
- Attention Quiz

READING QUIZ

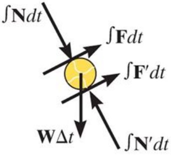
1. The internal impulses acting on a system of particles always ________.
   (a) equal the external impulses.
   (b) sum to zero.
   (c) equal the impulse of weight.
   (d) None of the above.
   ANS: __
2. If an impulse-momentum analysis is considered during the very short time of interaction, as shown in the picture, weight is a/an ________.

   (a) impulsive force.
   (b) explosive force.
   (c) non-impulsive force.
   (d) internal force.



ANS: __

APPLICATIONS

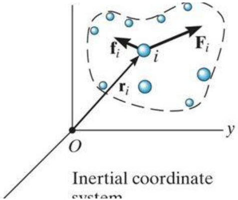


- As the wheels of this pitching machine rotate, they apply frictional impulses to the ball, thereby giving it linear momentum in the direction of $\boldsymbol{F}dt$ and $\boldsymbol{F}'dt$.
- The weight impulse, $\boldsymbol{W}\Delta t$ is ***very small*** since the time the ball is in contact with the wheels is very small.
- Does the release velocity of the ball depend on the mass of the ball?

APPLICATIONS(continued)

- This large crane-mounted hammer is used to drive piles into the ground.
- Conservation of momentum can be used to find the velocity of the pile just after impact, assuming the hammer does not rebound off the pile.
- If the hammer rebounds, does the pile velocity change from the case when the hammer doesn't rebound? Why?
- In the impulse-momentum analysis, do we have to consider the impulses of the weights of the hammer and pile and the resistance force? Why or why not?

PRINCIPLE OF LINEAR IMPULSE AND MOMENTUM FOR A SYSTEM OF PARTICLES (Section 15.2)



- For the system of particles shown, the internal forces $\boldsymbol{f}_i$ between particles always occur in pairs with _____ magnitude and _______ directions. Thus, ***the internal impulses sum to zero***.

- The linear impulse and momentum equation for this system only includes the impulse of _______ forces.

$$m_i \boldsymbol{v}_{i1} + \Sigma \int_{t_1}^{t_2} \boldsymbol{F}_i dt = m_i \boldsymbol{v}_{i2}$$

MOTION OF THE CENTER OF MASS

- For a system of particles, we can define a ”fictitious” center of mass of an aggregate particle of mass m_{tot}, where m_{tot} is the sum ($m_{\text{tot}} = \Sigma_i m_i$) of all the particles. This system of particles then has an aggregate velocity of

$$\boldsymbol{v}_G = \frac{\Sigma_i m_i \boldsymbol{v}_i}{m_{\text{tot}}}$$

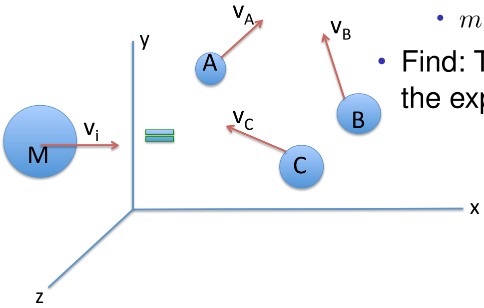
- The motion of this fictitious mass is based on motion of the center of mass for the system.
- The position vector

describes the motion of the center of mass.

CONSERVATION OF LINEAR MOMENTUM FOR A SYSTEM OF PARTICLES (Section 15.3)

- When the ***sum of external impulses*** acting on a system of objects is zero, the linear impulse-momentum equation simplifies to $\Sigma m_i \boldsymbol{v}_{i1} = \Sigma m_i \boldsymbol{v}_{i2}$.
- This equation is referred to as the ***conservation*** of ***linear momentum***. Conservation of linear momentum is often applied when particles _____ or _____. When particles impact, only ***impulsive forces*** cause a change of linear momentum.
- The sledgehammer applies an impulsive force to the stake. The weight of the stake is considered negligible, or ***non-impulsive***, as compared to the force of the sledgehammer. Also, provided the stake is driven into soft ground with little resistance, the impulse of the ground acting on the stake is considered ***impulsive forces***.

EXAMPLE I



- Given:
  - M = 100 kg, $\boldsymbol{v}_i = +20\mathbf{j}$ (m/s)
  - m_A = 20 kg, $\boldsymbol{v}_A = +50\mathbf{i} + 50\mathbf{j}$ (m/s)
  - m_B = 30 kg, $\boldsymbol{v}_B = -30\mathbf{i} - 50\mathbf{k}$ (m/s)
- Find: The velocity of fragment C after the explosion.

- Plan: Since the internal forces of the explosion cancel out, we can apply the conservation of linear momentum to the system.

EXAMPLE I (continued)

- Solution:

$$
\begin{aligned}
M\boldsymbol{v}_i &= m_A\boldsymbol{v}_A + m_B\boldsymbol{v}_B + m_C\boldsymbol{v}_C && (1)\\
100(20\mathbf{j}) &= 20(50\mathbf{i} + 50\mathbf{j}) + 30(-30\mathbf{i} - 50\mathbf{k}) \\
&\quad +50(v_{cx}\mathbf{i} + v_{cy}\mathbf{j} + v_{cz}\mathbf{k}) && (2)
\end{aligned}
$$

- Equating the components on the left and right side yields:

$$
\begin{aligned}
0 = 1000 - 900 + 50(v_{cx}), &\quad v_{cx} = -2\text{m/s} && (3)\\
2000 = 1000 + 50(v_{cy}), &\quad v_{cy} = 20\text{m/s} && (4)\\
0 = -1500 + 50(v_{cz}), &\quad v_{cz} = 30\text{m/s} && (5)
\end{aligned}
$$

- So $\boldsymbol{v}_c = ($ $)$ m/s immediately after the explosion.

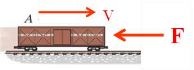
EXAMPLE II

- Given: Two rail cars with masses of m_A = 20 Mg and m_B = 15 Mg and velocities as shown.
- Find: The speed of the car A after collision if the cars collide and rebound such that B moves to the right with a speed of 2 m/s. Also find the average impulsive force between the cars if the collision place in 0.5 s.
- Plan: Use ***conservation of linear momentum*** to find the velocity of the car A after collision (all internal impulses cancel).Then use the ***principle of impulse and momentum*** to find the impulsive force by looking at only one car.

EXAMPLE II (solution)

- Conservation of linear momentum (x-dir):
  $m_A(v_{A1}) + m_B(v_{B1}) = m_A(v_{A2}) + m_B(v_{B2})$
  $20000(3) + 15000(-1.5) =$
  $(20000)v_{A2} + 15000(2)$
  $v_{A2} = \qquad\quad \mathrm{m/s}$

- Impulse and momentum on car A (x-dir):
  $m_A(v_{A1}) + \int(-F)dt = m_A(v_{A2})$
  $20000(3) - \int Fdt = 20000(0.375)$
  $\int Fdt = \qquad\quad \mathrm{Ns}$

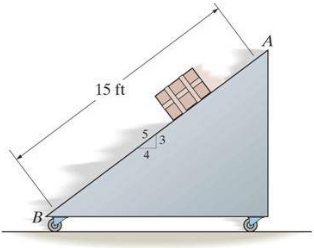


- The average force is
  $\int Fdt = 52500\,\mathrm{Ns} = F_{\mathrm{avg}}(0.5\mathrm{sec})$
  $F_{\mathrm{avg}} = 105\mathrm{kN}$

CONCEPT QUIZ

1. Over the short time span of a tennis ball hitting the racket during a player's serve, the ball's weight can be considered.
   (a) nonimpulsive.
   (b) impulsive.
   (c) not subject to Newton's second law.
   (d) Both (a) and (c).
   ANS: __
2. A drill rod is used with a air hammer for making holes in hard rock so explosives can be placed in them. How many impulsive forces act on the drill rod during the drilling?
   (a) None
   (b) One
   (c) Two
   (d) Three
   ANS: __

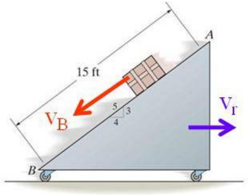
GROUP PROBLEM SOLVING



- Given: The free-rolling ramp has a weight of 120 lb. The 80 lb crate slides from rest at A, 15 ft down the ramp to B. Assume that the ramp is smooth, and neglect the mass of the wheels.

- Find: The ramp's speed when the crate reaches B.
- Plan: Use the energy conservation equation as well as conservation of linear momentum and the relative velocity equation (you thought you could safely forget it?) to find the velocity of the ramp.

GROUP PROBLEM SOLVING (continued)



- Energy conservation equation, potential = kinetic:
  $0 + 80\frac{3}{5}(15) = 0.5\frac{80}{32.2}(v_B)^2 + 0.5\frac{120}{32.2}(v_r)^2$
- To find the relations between v_B and v_r, use conservation of linear momentum:

$$(+\rightarrow)0 = \frac{120}{32.2}v_r - \frac{80}{32.2}v_{Bx} \qquad (6)$$

$$v_{Bx} = 1.5v_r \qquad (7)$$

- Since $\boldsymbol{v}_B = \boldsymbol{v}_r + \boldsymbol{v}_{B/r}$

$$-\boldsymbol{v}_{Bx}\mathbf{i} + \boldsymbol{v}_{By}\mathbf{j} = v_r\mathbf{i} + v_{B/r}(-4/5\mathbf{i} - 3/5\mathbf{j}) \qquad (8)$$

$$-v_{Bx} = v_r - (4/5)v_{B/r} \qquad (9)$$

$$v_{By} = -(3/5)v_{B/r} \qquad (10)$$

- Eliminating $v_{B/r}$ from Eqs. (9) and (10) and substituting Eq. (7) results in ____________

GROUP PROBLEM SOLVING(continued)

Then, ***energy conservation*** equation can be rewritten;

$$
\begin{aligned}
0 + 80\tfrac{3}{5}(15) &= 0.5\tfrac{80}{32.2}(v_B)^2 + 0.5\tfrac{120}{32.2}(v_r)^2 && (11)\\
0 + 80\tfrac{3}{5}(15) &= 0.5\tfrac{80}{32.2}[(1.5v_r)^2 + (1.875v_r)^2] \\
&\quad +0.5\tfrac{120}{32.2}(v_r)^2 && (12)\\
720 &= 9.023(v_r)^2 && (13)\\
v_r &= \quad \mathrm{ft/s} && (14)
\end{aligned}
$$

ATTENTION QUIZ

1. The 20 g bullet is fired horizontally at 1200 m/s into the 300 g block resting on a smooth surface. If the bullet becomes embedded in the block, what is the velocity of the block immediately after impact.

(a) 1125 m/s

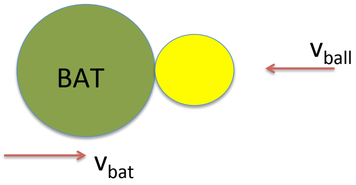
(b) 80 m/s

(c) 1200 m/s

(d) 75 m/s

ANS: __

2. The 200 g baseball has a horizontal velocity of 30 m/s when it is struck by the bat, B, weighing 900 g, moving at 47 m/s. During the impact with the bat, how many impulses of importance are used to find the final velocity of the ball?



(a) Zero
(b) One
(c) Two
(d) Three

ANS: __

IMPACT

Today's objectives: Students will be able to

1. Understand and analyze the mechanics of ***impact***.
2. Analyze the motion of bodies undergoing a collision, in both central and oblique cases of impact.

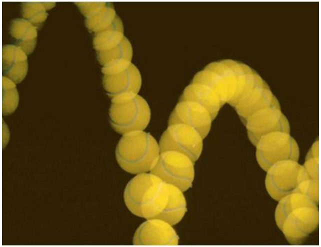
In-class activities:

- Reading Quiz
- Applications
- Central Impact
- Coefficient of Restitution
- Oblique Impact
- Concept Quiz
- Group Problem Solving
- Attention Quiz

READING QUIZ

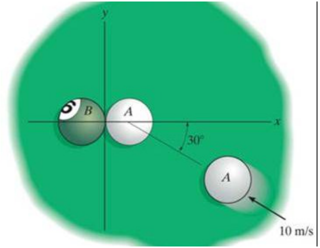
1. When the motion of one or both of the particles is at an angle to the line of impact, the impact is said to be
   (a) central impact.
   (b) oblique impact.
   (c) major impact.
   (d) None of the above.
   ANS: __
2. The ratio of the restitution impulse to the deformation impulse is called
   (a) impulse ratio.
   (b) restitution coefficient.
   (c) energy ratio.
   (d) mechanical efficiency.
   ANS: __

APPLICATIONS



- The quality of a tennis ball is measured by the height of its bounce. This can be quantified by the coefficient of restitution of the ball.
- If the height from which the ball is dropped and the height of its resulting bounce are known, how can we determine the coefficient of restitution of the ball?

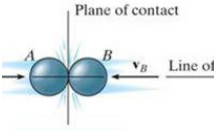
APPLICATIONS (continued)



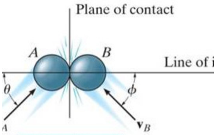
- In the game of billiards, it is important to be able to predict the trajectory and speed of a ball after it is struck by another ball.
- If we know the velocity of ball A before the impact, how can we determine the ____________________ of the velocity of ball B after the impact?
- What parameters do we need to know for this?

IMPACT (Section 15.4)

- Impact occurs when two bodies ***collide*** during a very ***short*** time period, causing large impulsive forces to be exerted between the bodies. Common examples of impact are a hammer striking a nail or a bat striking a ball. The line of impact is ***a line through the mass centers*** of the colliding particles. In general, there are two types of impact:



Central impact

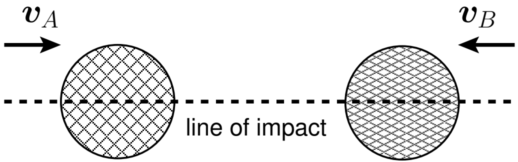


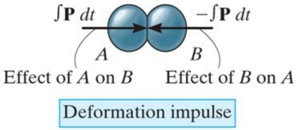
Oblique impact

- ______ impact occurs when the directions of motion of the two colliding particles are along the line of impact.
- ______ impact occurs when the direction of motion of one ***or*** both of the particles is at an angle to the line of impact.

CENTRAL IMPACT

- Central impact happens when the velocities of the two objects are along the line of impact (recall that the line of impact is a line through the particles' mass centers).





- Once the particles contact, they may ***deform*** if they are ***non-rigid***. In any case, energy is ***transferred*** between the two particles.
- There are two primary equations used when solving impact problems. The textbook provides extensive detail on their derivation.

CENTRAL IMPACT (continued)

- In most problems, the initial velocities of the particles, $(v_A)_1$ and $(v_B)_1$, are known, and it is necessary to determine the final velocities, $(v_A)_2$ and $(v_B)_2$. So the first equation used is the ***conservation of linear momentum***, applied along the line of impact.

$$(m_A v_A)_1 + (m_B v_B)_1 = (m_A v_A)_2 + (m_B v_B)_2$$

- This provides one equation, but there are usually two unknowns, $(v_A)_2$ and $(v_B)_2$. So another equation is needed. The ***principle of impulse and momentum*** is used to develop this equation, which involves the coefficient of ________, or e.

CENTRAL IMPACT (continued)

- The coefficient of restitution, e, is the ratio of the particles' relative separation velocity after impact, $(v_B)_2 - (v_A)_2$, to the particles' relative approach velocity before impact, $(v_A)_1 - (v_B)_1$. The coefficient of restitution is also an indicator of the ________ during the impact.
- The equation defining the coefficient of restitution, e, is

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1} = \frac{(v_{B/A})_{\text{after}}}{(v_{A/B})_{\text{before}}}$$

- If a value for e is specified, this relation provides the second equation necessary to solve for $(v_A)_2$ and $(v_B)_2$.

COEFFICIENT OF RESTITUTION

- In general, e has a value between zero and one. The two limiting conditions can be considered:
- Elastic impact (e = 1): In a perfectly _____ collision, no energy is lost and the relative separation velocity equals the relative approach velocity of the particles. In practical situations, this condition _____ be achieved.
- Plastic impact (e = 0): In a _____ impact, the relative separation velocity is zero. The particles __________ and move with a common velocity after the impact.
- Some typical values of e are:

Steel on steel:	$0.5 - 0.8$
Wood on wood:	$0.4 - 0.6$
Lead on lead:	$0.12 - 0.18$
Glass on glass:	$0.93 - 0.95$

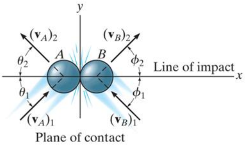
IMPACT: ENERGY LOSSES

- Once the particles' velocities before and after the collision have been determined, the energy loss during the collision can be calculated on the basis of the difference in the particles' _____ energy. The energy loss is

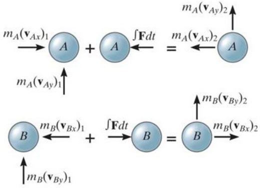
$$\Sigma U_{1-2} = \Sigma T_2 - \Sigma T_1$$

  where $T_i = \frac{1}{2} m_i (v_i)_2$ (______)
- During a collision, some of the particles' initial kinetic energy will be lost in the form of ___, _____, or due to localized __________.
- In a plastic collision (e = 0), the energy lost is a ________, although it does not necessarily go to zero. Why?

OBLIQUE IMPACT



- In an oblique impact, one or both of the particles' motion is at an angle to the line of impact. Typically, there will be four unknowns: the __________ and ________ of the final velocities.
- The four equations required to solve for the unknowns are:



1. Conservation of momentum and the coefficient of restitution equation are applied along the line of impact (x-axis):
$$m_A(v_{Ax})_1 + m_B(v_{Bx})_1 = m_A(v_{Ax})_2 + m_B(v_{Bx})_2$$
$$e = [(v_{Bx})_2 - (v_{Ax})_2]/[(v_{Ax})_1 - (v_{Bx})_1]$$
2. Momentum of each particle is conserved in the direction perpendicular to the line of impact (y-axis):
$$m_A(v_{Ay})_1 = m_A(v_{Ay})_2 \text{ and } m_B(v_{By})_1 = m_B(v_{By})_2$$

PROCEDURE FOR ANALYSIS

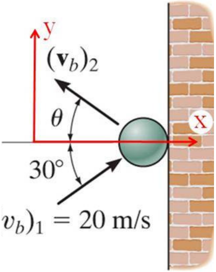
- In most impact problems, the initial velocities of the particles and the coefficient of restitution, e, are known, with the final velocities to be determined.
- Define the $x - y$ axes. Typically, the x-axis is defined ________________ and the y-axis is in the plane of contact ***perpendicular*** to the x-axis.
- For both central and oblique impact problems, the following equations apply along the line of impact (x-dir.):

$$\Sigma m(v_x)_1 = \Sigma m(v_x)_2 \text{ and}$$
$$e = [(v_{Bx})_2 - (v_{Ax})_2]/[(v_{Ax})_1 - (v_{Bx})_1]$$

- For oblique impact problems, the following equations are also required, applied __________ to the line of impact (y-dir.):

$$m_A(v_{Ay})_1 = m_A(v_{Ay})_2 \text{ and } m_B(v_{By})_1 = m_B(v_{By})_2$$

EXAMPLE

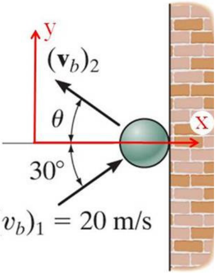


- Given: The ball strikes the smooth wall with a velocity $(v_b)_1 = 20$ m/s. The coefficient of restitution between the ball and the wall is $e = 0.75$.
- Find: The velocity of the ball just after the impact.

- Plan: The collision is an oblique impact, with the line of impact perpendicular to the plane (through the relative centers of mass). Thus, the coefficient of restitution applies perpendicular to the wall and the _________ of the ball is conserved along the wall.

EXAMPLE (Solution)

- Solve the impact problem by using $x - y$ axes defined along and perpendicular to the line of impact, respectively:



- The momentum of the ball is conserved in the y-dir:

$$m(v_b)_1 \sin 30° = m(v_b)_2 \sin\theta$$
$$(v_b)_2 \sin\theta = 10\,\mathrm{m/s} \qquad (1)$$

- The coefficient of restitution applies in the x-dir:

$$e = [0 - (v_{bx})_2]/[(v_{bx})_1 - 0]$$
$$\Rightarrow 0.75 = [0 - (-v_b)_2 \cos\theta]/[20\cos 30° - 0]$$
$$\Rightarrow (v_b)_2 \cos\theta = 12.99\,\mathrm{m/s} \qquad (2)$$

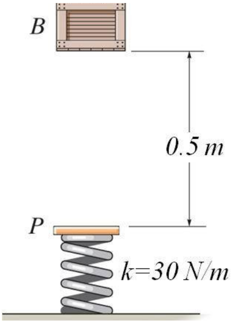
- Using Eqs. (1) and (2) and solving for the velocity and θ yields:

$$(v_b)_2 = (12.99^2 + 10^2)^{0.5} = 16.4\,\mathrm{m/s}$$
$$\theta = \tan^{-1}(10/12.99) = 37.6°$$

CONCEPT QUIZ

1. Two balls impact with a coefficient of restitution of 0.79. Can one of the balls leave the impact with a kinetic energy greater than before the impact?
   (a) Yes
   (b) No
   (c) Impossible to tell
   (d) Don't pick this one!
   ANS: __
2. Under what condition is the energy lost during a collision maximum?
   (a) $e = 1.0$
   (b) $e = 0.0$
   (c) $e = -1.0$
   (d) Collision is non-elastic.
   ANS: __

GROUP PROBLEM SOLVING

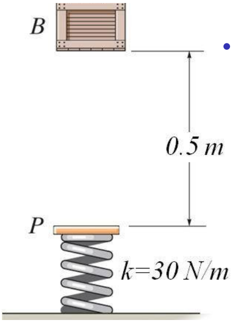


- Given: A 2 kg crate B is released from rest, falls a distance $h = 0.5$ m, and strikes plate P (3 kg mass). The coefficient of restitution between B and P is $e = 0.6$, and the spring stiffness is $k = 30\,\mathrm{N/m}$.
- Find: The velocity of crate B just after the collision.

- Plan:

1. Determine the speed of the crate just before the collision using projectile motion or an energy method.
2. Analyze the collision as a central impact problem.

GROUP PROBLEM SOLVING (continued)



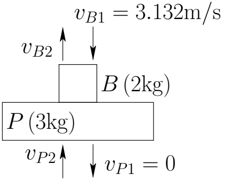
- Determine the speed of block B just before impact by using conservation of energy (why?).
- Define the gravitational datum at the initial position of the block (h_1 = 0) and note the block is released from rest ($v_1 = 0$):

$$\begin{aligned} T_1 + V_1 &= T_2 + V_2 && (15) \\ 0.5m(v_1)^2 + mgh_1 &= 0.5m(v_2)^2 + mgh_2 \\ 0 + 0 &= 0.5(2)(v_2)^2 + (2)(9.81)(-0.5) \\ v_2 &= 3.132\,\mathrm{m/s}(\downarrow) && (16) \end{aligned}$$

- This is the speed of the block just before the collision. Plate (P) is at rest, velocity of zero, before the collision.

GROUP PROBLEM SOLVING(continued)

- Analyze the collision as a central impact problem.



- Apply conservation of momentum to the system in the vertical direction $(+\uparrow)$:

$$\begin{aligned} m_B(v_B)_1 + m_P(v_P)_1 &= m_B(v_B)_2 + m_P(v_P)_2 \\ (2)(-3.132) + 0 &= (2)(v_B)_2 + (3)(v_P)_2 \end{aligned}$$

- Using the coefficient of restitution $(+\uparrow)$:

$$\begin{aligned} e &= [(v_P)_2 - (v_B)_2]/[(v_B)_1 - (v_P)_1] \\ 0.6 &= [(v_P)_2 - (v_B)_2]/[-3.132 - 0] \\ -1.879 &= (v_P)_2 - (v_B)_2 \end{aligned}$$

- Solving the two equations simultaneously yields

$(v_B)_2 = \qquad \text{m/s}\downarrow$ and $(v_P)_2 = \qquad \text{m/s}\downarrow$

- Both the block and plate will travel down after the collision.

ATTENTION QUIZ

1. Block B (1 kg) is moving on the smooth surface at $10\,\mathrm{m/s}$ when it squarely strikes block A (3 kg), which is at rest. If the velocity of block A after the collision is 4 m/s to the right, $(v_B)_2$ is

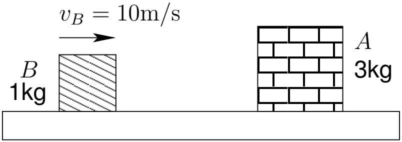
   (a) $2\,\mathrm{m/s} \rightarrow$

   (b) $7\,\mathrm{m/s} \leftarrow$

   (c) $7\,\mathrm{m/s} \rightarrow$

   (d) $2\,\mathrm{m/s} \leftarrow$

   ANS: __



ATTENTION QUIZ (continued)

2. A particle strikes the smooth surface with a velocity of $30\,\mathrm{m/s}$. If $e = 0.8$, $(v_x)_2$ is ____________ after the collision.

   (a) Zero

   (b) equal to $(v_x)_1$

   (c) less than $(v_x)_1$

   (d) greater than $(v_x)_1$

   ANS: __